## The Beauty of Turtle Graphics


 wan- pushind

The Beauty of Turtle Graphics
An illustrated Introduction

Author: Dr. Joachim Wedekind
Eschenweg 26
72076 Tübingen

The present work has been carefully prepared. Nevertheless, the author accepts no liability for the correctness of information, notes and tips, nor for any printing errors.

Website:
http://programmieren.joachim-wedekind.de/logo-classics/

2020 Tübingen
© Copyright for the complete works lies with the author
Graphic design and typesetting: Joachim Wedekind
Imprint: http://joachim-wedekind.de/impressum/

This work by Joachim Wedekind is under a Creative Commons License 4.0: Attribution, non-commercial, no modification. Permissions beyond this license can be obtained from the author.
(c) (i) $\Theta(\Theta)$

## Preface

Shortly after my professional entry into teaching technology and media didactics, I read for the first time in 1975 in an overview volume „Computer und Unterricht" (Eyferth et al., 1974) about the Logo project of Papert at MIT. Considered by the authors "as one of the most remarkable attempts of meaningful computer use in class according to its curicular and didactic objectives", they tried to prove this by means of typical program examples. These showed simple geometric line figures drawn with the "Turtle", a controllable robot
These pictures were and are typical for the use of Logo in a school context. They can be found in early publications on Logo (the manuals for different Logo versions and many introductory books). I was immediately attracted by the images and the possibilities for their creation. For me, they have a very unique aesthetic with a high recognition value in the corresponding publications.
This book therefore focuses on typical images created with the turtle graphic. In some examples, the proximity to works of early computer art (which I have included as homages to the artists), but also to well-known optical illusions (which I call "Opticals") becomes very obvious

Only basic programming concepts that are used to generate the respective images are briefly outlined. The book is therefore not an introduction to programming with Logo, but rather a picture book, which may inspire you to program these and comparable pictures yourself.

Tübingen, April 2020 Joachim Wedekind

By the way, one more thing: I apologize for the poor linguistic quality of this booklet. It is my own translation of the German original. In some sections I used the help of DeepL or Google Translate. Corrections and improvements are welcome.
Advertising on my own behalf: There are two books and related websites by me about Computer art and Opticals that give programming the appropriate status:
J. Wedekind (2018). Codierte Kunst. http://digitalart.joachim-wedekind.de/about-the-book// J. Wedekind (2019): Optische Täuschungen animieren für Dummies Junior.
http://opticals.joachim-wedekind.de/das-buch/
Content
Preface ..... I
Content ..... III
Introduction: Logo Classics ..... 1
Line Graphics4
Squares (I) ..... 10
Squares (II) ..... 12
Flags, spiders, swirls ..... 20
Rectangles, parallelograms ..... 24
pick random ... ..... 32
Polygons42
Squiggle, squaggle \& Co. ..... 54
Circles (I) ..... 62
Circles (II) ..... 64
Arcs ..... 72
Arcs - applications with variants ..... 80
Spirals ..... 88
Recursive spirals ..... 94
Dot spirals ..... 98
Arrow geometry ..... 104
Recursive trees ..... 108
Combination of known things ..... 114
Supersigns ..... 144
Moiré-patterns ..... 172
Stamping instead of drawing ..... 180
Outlook: Animation and Interaktion ..... 190
Literature ..... 193

## Introduction: Logo Classics

After years I rediscovered the book Mindstorms by Seymour Papert (Papert, 1980, 1982)¹. In this book he unfolds his view of the computer as a mental tool and how children can access this tool with the help of Logo.
Turtle graphics was introduced as a central component of Logo in 1970. With a few basic commands it is possible to create simple, but also complex appealing graphics².

"TURTLE GEOMETRY is a different style of doing geometry, just as Euclid's axiomatic style and Descartes's analytic style are different from one another. Euclid's is a logical style. Descartes's is an algebraic style. Turtle geometry is a computational style of geometry." (Papert, 1982, S. 55)
The creation of these images can be easily understood using code examples ${ }^{3}$. Those who enjoy abstract-geometric graphics will find a variety of suggestions on how to create appealing images using simple programming technology.

[^0]For me, the approach that underlies the turtle graphics is still best described by Seymour Papert himself in Mindstorms. Its basic principles are presented in a compact form and illustrated in Chapter 3: Turtle Geometry: A Mathematics Made for Learning. They will be illustrated here with interactive examples.
Note: In order to understand the examples, it is recommended to look at the complete code of the examples. The links to these can be found on the website Logo Classics. A click on the program name leads directly to the programs in the Snap! programming environment.

When Snap! is restarted, it opens with a predefined initial state: The stage has a certain width and height, is empty and has the color white The turtle sits in the middle of the stage in the shape of an arrow, points to the right and has the color black. All these properties can be easily changed later.
The turtle can understand commands in the so-called turtle language. These control their movement. With the command move, it moves in a straight line in its direction of view, so it changes its position, but not its direction. With the command turn it changes its direction of view, but not its position.

In the example opposite the movements of the turtle are described by such commands and thus a square is drawn:


## Line Graphics

Before the turtle can draw, its pen has to be lowered with pen down. Only then does it leave a trace. The pen can be raised again with pen up.

Many different basic patterns are already possible with just move and turn and pen down and pen up.


Iterations: The computer is particularly good at performing frequently recurring, dull but necessary actions patiently and quickly. For this purpose, Snap! includes the repeat block, whose enclosed commands are executed as often as specified.


If the basic patterns are combined and multiplied in the repeat loops, many attractive line graphics can already be created.


〈＜＜＜〉＞＞＞＞



## Squares (I)



In many introductions to programming, the first example of a simple sequence of commands is the sentence Hello World on the screen ${ }^{4}$. For me, drawing a square is in a way the Hello World in the languages of the Logo family! The square can be used to illustrate several important concepts:

Repetitions: In the repeat block, its commands will be executed as many times as specified, i.e. four times for the square

```
repeat 4
move 100 steps
turn }90\mathrm{ degrees
```

Expandability and reusability: The turtle language can be expanded through your own procedures. These command sequences can then be used again and again. In the procedure square, the commands for drawing the four sides of the same length and rotating the turtle by 90 degrees are combined


## Squares (II)

Variables: All kinds of data (numbers, texts, images, etc.) can be stored in variables. The computer will find the data safely, can use them again and again in the program and also change them.
The length of the sides is passed to the square procedure as input parameter length. The squares can now be drawn in any desired size.


Simplification through repetitions: The reuse of procedures in repetition loops also allows more complex figures; so below for nested squares and diamonds, controlled via n_squares, delta and angle.



## length 50 <br> length 50

length 250
length 500




Flags, spiders, swirls

Opticals: Scintillating Grid


Reusability and location neutrality: The advantage of procedures is their reusability in new contexts. Of course, this also applies to the square procedure. Local neutrality is important, i.e. the return of the turtle to its starting point with its original direction of vision.
A procedure flag_square draws the flagpole of length flagpole and then the square with the side length length. Finally, the turtle is always moved to the middle of the stage.

Thus, simple structures can easily be combined into more complex structures. This applies to spiders and swirls (on the following double page).


swirl


Rectangles, parallelograms
Modifications and extensions: Procedures can be taken as a starting point for new uses. For example, the procedure square can easily be changed to a procedure rectangle, to which the width and height of the rectangle are passed as values a and b, respectively:


The procedure rectangle can in turn be extended to a procedure parallelogram if the angle beta is passed in addition to the side lengths (the angle alpha results in the parallelogram as 180 -beta).


The parallelogram procedure now contains the initial procedures as special cases, namely the rectangle procedure with beta $=$ alpha $=90$ and the square procedure with additional $\mathrm{a}=\mathrm{b}$ !

parallelograms





## pick random ..

In the previous pictures, everything was clear and therefore predictable, determined by the program commands. Randomness can provide for moments of surprise in the pictures, because all characteristics of the turtle can easily be changed randomly. The basis is the random function pick random 1 to 10, which provides evenly distributed random numbers between 1 and 10 .

The range for the random numbers can be freely selected. With such limits, then it is spoken of guided randomness. For example,

```
set x to pick random -400 to 400
set y to pick random -200) to 200
```

is used to determine the random positions of the turtle. The picture on the top right shows accordingly the rather even distribution of points.
Chance can change everything! That affects, among other things

- the step length with move,
- the position with go to,
- the angle with turn,
- the pen thickness with set pen size to
- or the color with set pen color to.





 Wthomat余


Homage à Nees: Bunch of lines


Homage à Nees: Corner lines



Homage à Franke: Squares


Homage à Nees: Gravel



Polygons
The generalization of programs or procedures is the step from the familiar to the unknown. A small change in the procedure square makes this clear. Just by introducing a parameter n, which determines the number of corners of a polygon, instead of squares we can draw any regular polygon in a new procedure polygon with sides of equal length and equal angles.


Compared to the procedure square, only the angle of rotation changes in addition to the input values. With $360 / n$ it is ensured that the angle sum in the polygon always results in 360 degrees.

Interesting: In all regular polygons the turtle turns 360 degrees. Papert (Papert, 1982. p. 76) calls this The Total Turtle Trip Theorem:
„If a Turtle takes a trip around the boundary of any area and ends up in the state in which it started, then the sum of all turns will be 360 degrees."


## nested polygons




## Opticals: Ambiguous Figure Necker Cube

 (6 hexagons, angle 60)


둔 둔
분군 를





## Squiggle, squaggle \& Co.

For the complete Total Turtle Trip Theorem there is an important generalization. Even with open polygons, the turtle ends in the initial state when the sum of all right and left rotations is again 360 degrees or a multiple thereof.

Brian Harvey introduced an example with the funny name squiggle (Harvey, 1982, p. 186 ff.). The always same sequence of pairwise move and turn movements can be easily and flexibly implemented with a procedure designs and by summarizing the movement data in a list.

Lists are fields in which numbers and/or texts, lists and even program code can be saved and called up again.

The movement data for squiggle and two other
variants called squaggle and squoggle then result in the following lists:

> set squiggle * to list $100 \quad 135 \quad 40 \quad 120 \quad 60 \quad 15$ 4 *
> set squaggle ~ to list $5 0 \longdiv { 1 5 0 } 6 0 1 0 0 3 0 9 0$ it
> set squoggle * to list $50 \quad 70 \quad 10 \quad 160 \quad 35 \quad 58$ *

With designs, similar variants can now be easily drawn.

But it is only through repetition and by changing the values for the movements that the actually interesting patterns are created.


In addition there is a similar example by Hal Abelson (1983, p. 49) with the simple name design.
set design * to list $1009010090 \quad 5090 \quad 509010090 \quad 25 \quad 90 \quad 2590 \quad 50 \quad 0 \quad$ +




## Circles (I)

When drawing figures, Papert liked to let the children play turtle themselves. He speaks of bodysyntonic learning or ego-syntonic learning to make clear the relationship between geometric operations and physical experience.

A prime example of this is the drawing of a circle: from the repeated sequence of body movements "take a small step forward, turn a little" a circle is actually formed.

From the verbal description it is only a small step to the program. A repeat loop with the repaet block shows the expected result.


If circles of different sizes are to be drawn, this can be achieved in very different ways. One possibility is a procedure circle, where the radius of the circle is determined by the number of steps of the turtle:


The procedure circle is thus very similar to the procedure polygon, whereby here the angle is kept constant at one degree.

## Circles (II)

With a little mathematics, the circles can be determined more precisely. From the formula for the circumference and the mathematical constant $\pi$ the required step sizes of the turtle can be calculated using the radius: $\mathbf{U}=\mathbf{2} \boldsymbol{\pi} \mathbf{r}$. This results in the procedure circle radius:

```
circle radius-r +
ryeat 360
move (2) \ Pi < < 300
turn ct1 degrees
```

When drawing circles on paper, their position is determined with the ruler, and their radius with the compass around the selected center. A procedure circle araund that corresponds to this is somewhat more extensive:

- First the turtle is sent to the selected point $[x, y]$.
- From this point it is sent with radius to a starting point on the circle and
- draws the circle from there.
- Finally, the turtle is sent back to the starting point






Homage à Delaunay: Rhythm


Homage à Calder

## Arcs

Circles are the prime example of the relationship between geometric operations and physical experience, as described by Papert. They can also be used to show that omission can create new diversity. Thus circles can be turned into arcs by limiting steps and rotations.
As with circles, there are again several ways to create them, starting from the center point.

Drawing with a selected center point is also obvious for circular arcs. A distinction must be made between arcs that start from a point and arcs that go around a point. These procedures are now somewhat more extensive:



```
C-4tgenoun= iun
[-1 m-LEnici meve
```



```
f=+"
-k+"-1.
1-1, (a)+%m
- +&-4
4-+matho=
=-m
#
```




```
man ten yon
C-3)
(-7)N
C+\infty
MOM=O
```



By combining them many new interesting figures are created.



shifted arcs $180^{\circ}$


## Arcs - applications with variations

Arcs can be used as the basis for more complex figures. Leaves can be drawn very easily, which in turn can be combined into flowers.

| tpetal_outline size |
| :---: |
| arc with size 60 |
| turn \& 120 degrees |
| arc with size 60 |
| turn $¢ \subset 120$ degrees |


| Mower_putine + n_leaves + size in |  |
| :---: | :---: |
| ror $i=0$ to neleavas |  |
| petal_outline size |  |
| turn © 360 / n_leaves | degrees |

With the fill command, colored filled petals can also be created and used in appropriate combinations.

Further variations are possible, if one distinguishes between right- and left-turning arcs (which internally differ only by the commands turn 1 degrees or turn -1 degrees). Quite different waves can result.

```
* wave number , \overline{n}}\mathrm{ +radius + r}\mathrm{ tangle + }\overline{w}
Neptat in
arc_right ir w
arclet! + I w
|
```


## leaf \& flower (arcs $120^{\circ}$ )


waves $\left(\operatorname{arcs} 45^{\circ}<\right.$ variations $<180^{\circ}$ )




## flowers (arcs $120^{\circ}$ )



## waves (arcs $120^{\circ}<$ variations < 200 ${ }^{\circ}$ )


amoeba (arcs $45^{\circ}<$ variants $<120^{\circ}$ )



Homage à Nees: $\operatorname{Arcs}\left(\operatorname{arcs} 90^{\circ}-360^{\circ}\right)$

86


Homage à Sýkora: Lines $\left(\operatorname{arcs} 30^{\circ}-120^{\circ}\right)$

## Spirals

If in the regular polygons the lengths of the sides and the angles between the sides become variable, we get spirals. There are several possibilities for this, which result in interesting differences.
Similar to the procedure polygon, the turtle walks a distance and rotates by an angle after each step. However, in the new procedure spiral, the distance is increased by a factor delta d each time (upper right picture). So that this procedure does not continue endlessly, a limit g must be set for the distance. If the condition \| > g returns the value true, this limit is exceeded and the procedure is aborted.


```
mest until | >0,
move I stape
tumit W degrem
chanye (1 - by d)
```

Instead of the distance, the angle of rotation can of course also be increased. To do this, change the command change I by d to change w by d.

```
change w * by d
```

The result is then a completely different one (picture below right). The curve is a clothoid (also known as Cornu-Spiral oder Euler-Spiral), where the curvature grows linearly with the path length.




## Recursive spirals

Recursion as a form of repetition is often introduced with spirals. Recursive procedures can contain one or more self-calls. The principle is particularly powerful because (also modified) parameters can be passed in this call.
A limit g must be specified so that the calls do not continue indefinitely. If the corresponding condition returns the value true, this limit is exceeded and the calls therefor are ended. Depending on the location of the self call within the procedure, three types of recursion are distinguished:
initial recursion:

- termination condition
- self-call of the procedure
- instruction(s)
centric recursion
- termination condition
- instruction(s)
- self-call of the procedure - instruction(s)
tail recursive
- termination conaition
- instruction(s)
- self-call of the procedure

The process is easy to follow on the spirals.




## Dot spirals

The basic structure of spirals often becomes much more visible if not only the lines between the spiral points are drawn, but their starting (and end) points.
Dots of a certain pen thickness are drawn by not telling the turtle the number of steps for the move steps command and thereby "stepping on the spot" as it were. A yellow dot with a thickness of 60 then arises as follows:


Small changes in the characteristics of the spirals, such as the length, the angle or the change delta, can cause large changes in the point structures. If this is combined with random changes, completely different structures are created again.




## Arrow geometry

The turtle also has sensors. This allows different values to be measured and further processed:

- distance to center/mouse-pointer/object provides the distance (in pixels) from the current position of the turtle to the center of the image, to the mouse pointer or to another object to be specified
- direction to center/mouse-pointer/object provides the direction (in degrees) from the current position of the turtle to the center of the image, to the mouse pointer or to another object to be specified.
- In the same way, further properties can be recognized and measured. For example, hue, saturation ... at delivers the color values below the mouse pointer or another object to be specified.

This can be used to create spirals in a completely different way; this is called arrow geometry

Starting position is a point on a circle around a given center (blue in the picture on the top right). For this point, then the tangent of the circle can be drawn (perpendicular to the distance line to the centre). On this tangent the turtle is moved by length. At its new position, the tangent is drawn again and the turtle is moved by length + delta.



## Recursive Trees

Drawing trees is a popular example of the use of recursion in turtle graphics. One reason for this is that when drawing a tree, there are always recurring sequences:

1. First the trunk is drawn.
2. A left branch is drawn at a branching point.
3. A right branch is drawn at the same junction
4. Important: at the end the turtle returns to the starting point.

If the branches are each used as the starting point for further ramifications, a complex branching structure is created, i.e. the tree with branches.

In the recursive version, therefore, move length steps when drawing the branches is replaced by the recursive call tree recursive length depth. In the figure on the right the results for a depth of 1 to 4 are shown. If depth $=0$ returns the value true, drawing is aborted.

Also the length and the angle at which the branches branch off can be changed depending on the depth. Combined with line thickness and colour, the results then resemble very real trees of certain species. Other values result in abstract figures with area-filling patterns






## Combination of known things

With the programs and procedures shown so far, points, lines, polygons, circles, arcs, spirals, and recursive trees could be drawn. Thus, a powerful figure construction kit is already available. In a further step these elements can be combined with each other ${ }^{5}$. This results in new structures again:

1. lines on circles
2. polygons on circles
3. polygon combinations
4. circle combinations
5. recursive triangles, squares, polygons
6. recursive polygon combinations
7. recursive circles

[^1] 114





## squares on a circle



rectangles on a circle

circles on a circle

lines on circles




Homage à Riley: Blaze
polygon combinations


## $\square$



polygon combinations










## Supersigns

The principle of the following examples is the rotation and mirroring of lines, arcs or polygons. This way new "supersigns" can be created. By this we mean perceived entireties composed of the elementary signs.
Then, by repeating and combining the new supersigns in a grid, interesting endless patterns can be created. Such periodic repetitions are often found in decorative patterns.

For the following applications, four similar graphic elements are created down_left, upper_left, down_right, and upper_right:

1. three corners
2. (diagonal) lines
3. triangles
4. Escher triangles ${ }^{6}$
5. bars
6. arcs

The procedures for the graphic elements are internally always structured in the same way. The example down_left shows that simple commands for moving the turtle are sufficient.

${ }^{6}$ The example can be found in one of his early workbooks, in which he experimented with such patterns (Schattschneider, 1990, p. 45). 144

three corners







湤同|



 VA V V





Escher-triangles
 $\mathfrak{N} \mathbb{N} \mathbb{N} \mathbb{N} \mathbb{N} \mathbb{N}$ SONOMAMANANABM SNONMNONONONON




bars
\& क क क क क क क क क क
 R $\}$





arcs





## Moiré-patterns

By superimposing the graphic elements already known (lines, circles, points), surprising optical effects can be achieved (this is called the Moiré effect). Their creation is very simple ${ }^{7}$.
The procedure is the same throughout (the blue dot in the graphics on the right always marks the center of the image):

- A first basic pattern is created with an element from the figure construction kit. This pattern forms the background (in the picture on the right, lines, circles, points and again lines from top to bottom).
- Depending on the desired effect, one or more additional patterns can be created and superimposed on the first pattern.
- The second pattern is usually slightly offset and/or rotated compared to the first one (in the picture on the right in the middle, lines, circles, points and again circles, slightly shifted from top to bottom).

The results can be found in the image on the right in the third column on the far right.

Moiré patterns are important in various technical areas. But they have also been used in painting since Op-Art.
The combination of different shapes in different colours and sizes often produces surprising results ${ }^{8}$.

[^2]
| man m m m m m m m m m m m m m m m (x) man |x|| | | | | | | | | | | | - | - | - - - | - | ค












Homage à Biasi: Lot Nr. 639


Homage à Oxenaar: 45 Cent Stamp

## Stamping instead of drawing

There are useful functions in the programming environment Snap! that can greatly simplify the drawing of complex graphics. In all previous projects, for example, the graphic elements were created by moving the turtle with the pencil down. However, it is also possible to make the turtle - in the form of its costume - into a graphic element itself,

For each graphic created, a context menu can be opened by rightclicking on the stage, in which such a costume is created from the graphic by selecting the option pen trails.


A costume list can be found in the program area under the tab Costumes.
The Looks category provides commands for customizing the costumes. The command switch to costume replaces the current turtle costume with the selected costume. next costume replaces the current costume with the next costume from the Costumes list. By set size to $x$ $\%$, the size of the costume can be changed in percent, with change size by $x$ by $\times$ pixels.

The important thing now is that with stamp the selected costume can be "stamped" at the current position of the turtle. It remains there, even if the turtle is moved further.

This makes it possible to multiply complex graphics more easily and significantly faster than with the corresponding procedures in repeat-loops. The following pictures can show this as an example.

squares


Homage à Mohr: P-105
lines

lines and points


Homage à Le Parc: Rotations


Homage à Le Parc: Dots






## Outlook: Animation and Interaction

You can find the complete code of the examples on the website Logo Classics. There, a click on the program name leads directly to the programs in the Snap! programming environment.
Some of the programs differ somewhat from the sequence shown in the text. This is because I have prepared the programs - as far as reasonable for animation. Furthermore these programs are interactive, i.e. they can be controlled very easily by the user. The procedure is always the same and easy to understand:

The animation principle is very simple and corresponds to the traditional stop-motion technique. Each image is drawn by the computer. The current image is continuously replaced by a new image with updated properties (such as color, length, angle, etc.). So it is the principle of paint - wipe paint ... If this happens fast enough, we get a flicker-free, animated image

| Porever | The animation takes place in an endless loop (forever). <br> clear |
| :--- | :--- |
| First the stage is erased (clear), followed by the <br> commands required to draw the picture. The warp <br> command is necessary because it ensures that the |  |
| draw | graphic output only takes place when the image internally <br> is completely drawn. This makes the image output more <br> fluid! |
| whatever |  |
| you | Sliders are provided for the characteristic values so that <br> the viewers can control the images interactively. If values |
| color | are changed, the image is updated with these values <br> during the next repeat loop. To avoid undesired effects, |
| color 100 | sensible minimum and maximum values should be <br> preset for the parameters. |

In the concrete example on the right-hand side, the number of corners and their side length are set dynamically for the polygons (which of course can only be displayed statically here)



## Literature

Abelson, H. (1982). Logo for the Apple ][. New York: McGraw-Hill.
Eyferth, K., Fischer, K., Kling, U., Korte, W., Laubsch, J., Löthe, H., Schmidt, R., Schulte, H. \& Werkhofer, K. (1974). Computer im Unterricht. Formen, Erfolge und Grenzen einer Lerntechnologie in der Schule. Stuttgart: Klett

Feurzeig, W. (2010). Toward a Culture of Creativity: A Personal Perspective on Logo's Early Years and Ongoing Potential. International Journal of Computers for Mathematical Learning, Vol. 15, 3, pp. 257-265

Harvey, B. (1997). Computer Science Logo Style. V. 1: Symbolic Computing Cambridge: MIT Press. (Download des Buches:
https://people.eecs.berkeley.edu/~bh/v1-toc2.htm
Nicolai, C. (2010). moiré index. Berlin: Die Gestalten Verlag
Papert, S. (1980). Mindstorms: children, computers, and powerful ideas. New York:
Basic Books. (Download:
http://worrydream.com/refs/Papert\ -\ Mindstorms\ 1st\ ed.pdf)
Schattschneider, D. (1990). Visions of Symmetry. New York: Freeman and
Company,

## Photo credits:

Book cover Mindstorms (p. 1): Photo J. Wedekind
Floor turtle (p. 1):
http://cyberneticzoo.com/cyberneticanimals/1969-the-logo-turtle-seymour-papert-marvin-minsky-et-al-american

All other pictures are screenshots from the respective programs or graphics generated with them.

 $t \mathrm{~T}$ with hen $\xrightarrow[y]{2}$

W+





 $+i+$


[^0]:    ${ }^{1}$ It is still worth reading, especially today, when it is considered when and in what form computer science lessons will be introduced in schools. The original English version of the book can be downloaded free of charge.
    ${ }^{2}$ A turtle-like robot was developed for the graphic output: the Yellow Turtle (Feurzeig, 2010). This vehicle could move and rotate, and raise and lower a pen. Lines can be drawn on a paper underneath. The robot was later replaced by a small symbol with a direction indicator on the screen.
    ${ }^{3}$ The examples are implemented with the visual programming environment Snap!, quasi a "granddaughter" of Logo. The graphics are interactive; their characteristics can be changed with sliders.

[^1]:    ${ }^{5}$ So since known elements are used, I will not print snippets of code in this section.

[^2]:    ${ }^{7}$ Static images are already suitable for presentation; however, they are particularly appealing to viewers if they run as program-controlled animation or, ideally, can be influenced interactively themselves (by manually shifting the patterns).
    ${ }^{8}$ With the moiré index, Carsten Nicolai has presented an entire book in which he carries out precisely such systematic transformations of basic elements in grids. Pretty much anything can be recoded according to the principle presented here.

